January 6th, 2018 Mock (Practice) AMC 12

Welcome!

$2018 = (10 + 987) \times (6 - 5) + 4 \times 3) \times 2 \times 1$

$2018 = 10 \times 9 + 8 \times 7 - 6 + 5^4 \times 3 + 2 + 1$

$2016 = 2^5 \cdot 3^2 \cdot 7^1$
2017 = prime number
$2018 = 2 \cdot 1009$
$2019 = 3 \cdot 673$
$2020 = 2 \cdot 2 \cdot 5 \cdot 101$
…
2027 = prime

2017 is a prime number. The next prime number after 2017 is 2027.

2018 is the sum of two squares $2018 = 43^2 + 13^2$

1009 is the sum of two squares $1009 = 15^2 + 28^2$

Daniel Plotnick

1. Let Q be the sum of all integers p satisfying the inequality $\frac{9p^2}{\left(\sqrt{3p+1}-1\right)^2} < 3p + 10$?

a) 0 b) 1 c) 3 d) 6 e) 10

2. Given that a, b, and c are positive even integers which satisfy the equation a + b + c = 2018. How many solutions does the equation have?

a) 403200 b) 507528 c) 508536 d) 510549 e) 2035153

3. Given a sequence such that $A_0 = 2$, $A_1 = -1$ and $A_{n+1} = 2A_n - A_{n-1}$, where $n \geq 1$, determine the value of A_{2018}.

a) 6049 b) --6049 c) 0 d) 6052 e) -6052

4. Find the minimum value of $\sqrt{x^2 - 4x + 13} + \sqrt{x^2 - 14x + 130}$.

a) $6\sqrt{13}$ b) 13 c) $\frac{13}{2}$ d) 14 e) $\sqrt{143}$

5. Suppose for real numbers $x_1, x_2, \ldots\ x_{2018}$ the following equation holds:

$$\sqrt{x_1 - 1} + \sqrt{x_2 - 1} + \cdots + \sqrt{x_{2018} - 1} = \frac{1}{2}(x_1 + x_2 + \cdots + x_{2018})$$

Find the value of $x_1 + x_2 + \cdots + x_{2018}$.

a) 2018 b) $1009\sqrt{5}$ c) $2018\sqrt{2}$ d) 3027 e) 4036

6. Given that a, b, and c are positive integers, and a + b = 2018, c – a = 2017, and a < b. Find the greatest possible value of a + b + c.

a) 4036 b) 5043 c) 5044 d) 5045 e) 6052

7. Let positive integers, T, satisfy the condition that the product of the digits of $T = T^2 - 11T - 23$. Find the number of such numbers T.

a) 0 b) 1 c) 6 d) 11 e) 23

8. Let x, y, and z > 1, p > 0, $\log_x p = 18$, $\log_y p = 21$, *and* $\log_{xyz} p = 9$. Find the value of $\log_z p$.

a) 126 b) 189 c) 378 d) $\frac{126}{13}$ e) $\frac{378}{13}$

9. Let a_n and b_n be the x-intercepts of the quadratic equation

$$y = n(n-1)x^2 - (2n-1)x + 1$$

where n is an integer greater than 1. Find the value of

$$a_2b_2 + a_3b_3 + \ldots + a_{2018}b_{2018}$$

a) $\frac{2017}{2018}$ b) $\frac{2018}{2019}$ c) $\frac{2018}{2017}$ d) $\frac{2019}{2018}$ e) 1

10. Let n be a positive integer and n < 2019. If $n^{2018} - 1$ is divisible by $(n-1)^2$, find the maximum value of n.

a) 2 b) 3 c) 1009 d) 1010 e) 2018

11. We will call a number good if the binary representation of a positive integer has the following properties:
1) The number of digits is 11. 2) The number of 1s is 7 and number of 0s is 4.
For example, 2018 is a "good" number. 2018 = 11111100010
Find the sum of all "good" numbers.

b) 214936 b) 322455 c) 386946 d) 429876 e) 506716

12. Given that the equation $a^2x^2 - (4a - 3a^2)x + 2a^2 - a - 21 = 0$ (where a > 0) has at least one integral root. Find the sum of all possible values of a.

a) 1 b) 3 c) 4 d) 7 e) 11

13. How many pairs of distinct integers between 1 and 2018 inclusive have their product as multiples of 5?

a) 650845 b) 731848 c) 812851 d) 813254 e) 1140574

14. Given that x is a real number, find the least value of $\sqrt{2x(x+5)(x+6)(x+11)+2018}$

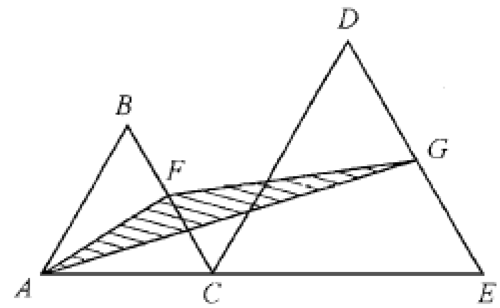
a) $22\sqrt{2}$ b) $24\sqrt{2}$ c) $26\sqrt{2}$ d) $28\sqrt{2}$ e) $30\sqrt{2}$

15. If x is real number, find the minimum value of the following function:

$f(x) = |x-1| + |x-2| + \ldots + |x-2018|$.

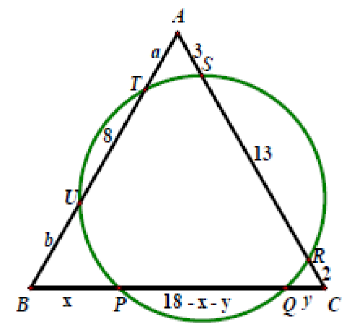
a) 1009^2 b) $\frac{2018^2}{2}$ c) $\frac{2018\times2019}{2}$ d) $\frac{2018\times2019}{4}$ e) $\frac{2018^2}{4} - 1$

16. In the figure at right, C lies on AE, ΔABC and ΔCDE are equilateral triangles, F and G are the mid-points of BC and DE respectively. If the area of ΔABC is 24 cm^2, the area of ΔCDE is 60 cm^2, and the area of ΔAFG is X cm^2, find the value of X.



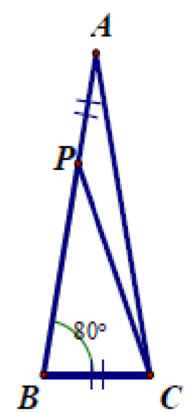
a) 12 b) 15 c) $12\sqrt{3}$ d) $8\sqrt{3}$ e) $\frac{25}{2}$

17. In figure at right, ABC is an equilateral triangle intersecting the circle at six points P, Q, R, S, T and U. If $AS = 3$, $SR = 13$, $RC = 2$ and $UT = 8$, find the value of $BP - QC$.



a) 2 b) 3 c) $2\sqrt{3}$ d) 4 e) 5

18. In the figure at right, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 80°$. If P is a point on the AB such that $AP = BC$, $\angle ACP = k°$, find the value of k.

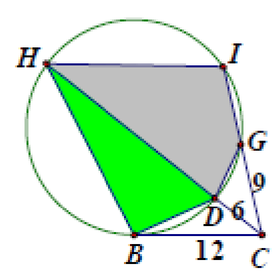


a) 6 b) 8 c) 9 d) 10 e) 12

19. Let $y = \log_{1400}\sqrt{2} + \log_{1400}\sqrt[3]{5} + \log_{1400}\sqrt[6]{7}$, find the value of y.

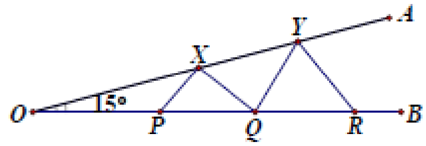
a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$ e) $\frac{1}{6}$

20. In the figure at right, B, H and I are points on the circle. C is a point outside the circle. BC is tangent to the circle at B. HC and IC cut the circle at D and G respectively. It is given that HDC is the angle bisector of $\angle BCI$, $BC = 12$, $DC = 6$ and $GC = 9$. Find the ratio of the area of ΔBDH to the area of quadrilateral $DHIG$.



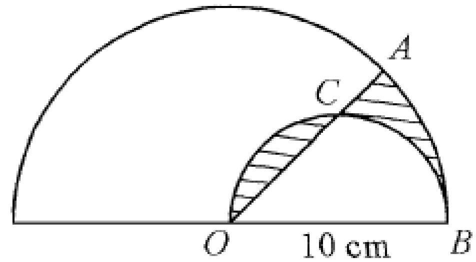
b) $\frac{2}{5}$ b) $\frac{24}{55}$ c) $\frac{3}{5}$ d) $\frac{36}{55}$ e) $\frac{8}{11}$

21. In the figure at right, $\angle AOB = 15°$. X, Y are points on OA, P, Q, R are points on OB such that $OP = 2$ and $OR = 6$. If $s = PX + XQ + QY + YR$, find the least value of s.



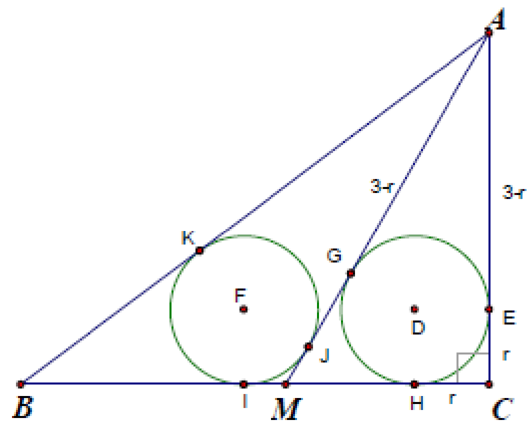
a) $\sqrt{12}$ b) 4 c) $3\sqrt{3}$ d) $2\sqrt{7}$ e) 5

22. In the figure at right, O is the center of the bigger semicircle with radius 10 cm, OB is the diameter of the smaller semicircle and C is the midpoint of arc OB and it lies on the segment OA. Let the area of the shaded region be K cm2, find the value of K.



a) 8π b) 25 c) $\frac{25}{2}(\pi - 1)$ d) $\frac{25}{2}(\pi - 2)$ e) $\frac{25}{4}(2\pi - 1)$

23. As shown in the figure at right, $AC = 3$, $BC = 4$ and $\angle C = 90°$. M is a point on BC such that the incircles in ΔABM and ΔACM are equal. Find the length of AM.
Hint: Let D, F be the centers of the circles, with radii r, let the points of contact be E, G, H, I, J, K as shown. Let $AM = x$, $AB = 5$, $CH = CE = r$.



a) $\sqrt{10}$ b) $2\sqrt{3}$ c) $\frac{7}{2}$ d) 4 e) $2\sqrt{5}$

24. Supposed that there are 6 different values of a real number x such that $\left||x^2 - 6x - 16| - 10\right| = y$. Find y.

a) 0 b) 8 c) 10 d) 15 e) 18

25. In the figure at right, X, Y and Z are points on BC, CA and AB of ΔABC respectively such that $\angle AZY = \angle BZX$, $\angle BXZ = \angle CXY$ and $\angle CYX = \angle AYZ$. If $AB = 10$, $BC = 6$ and $CA = 9$, find the length of AZ.

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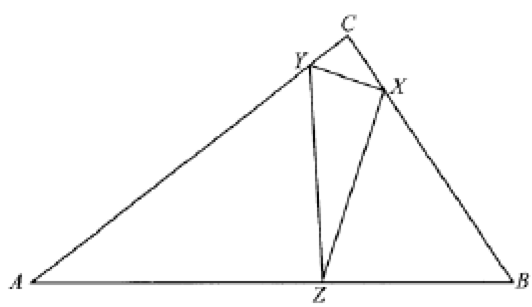

a) $\frac{25}{4}$ b) $\frac{26}{4}$ c) $\frac{27}{4}$ d) 7 e) $\frac{29}{4}$