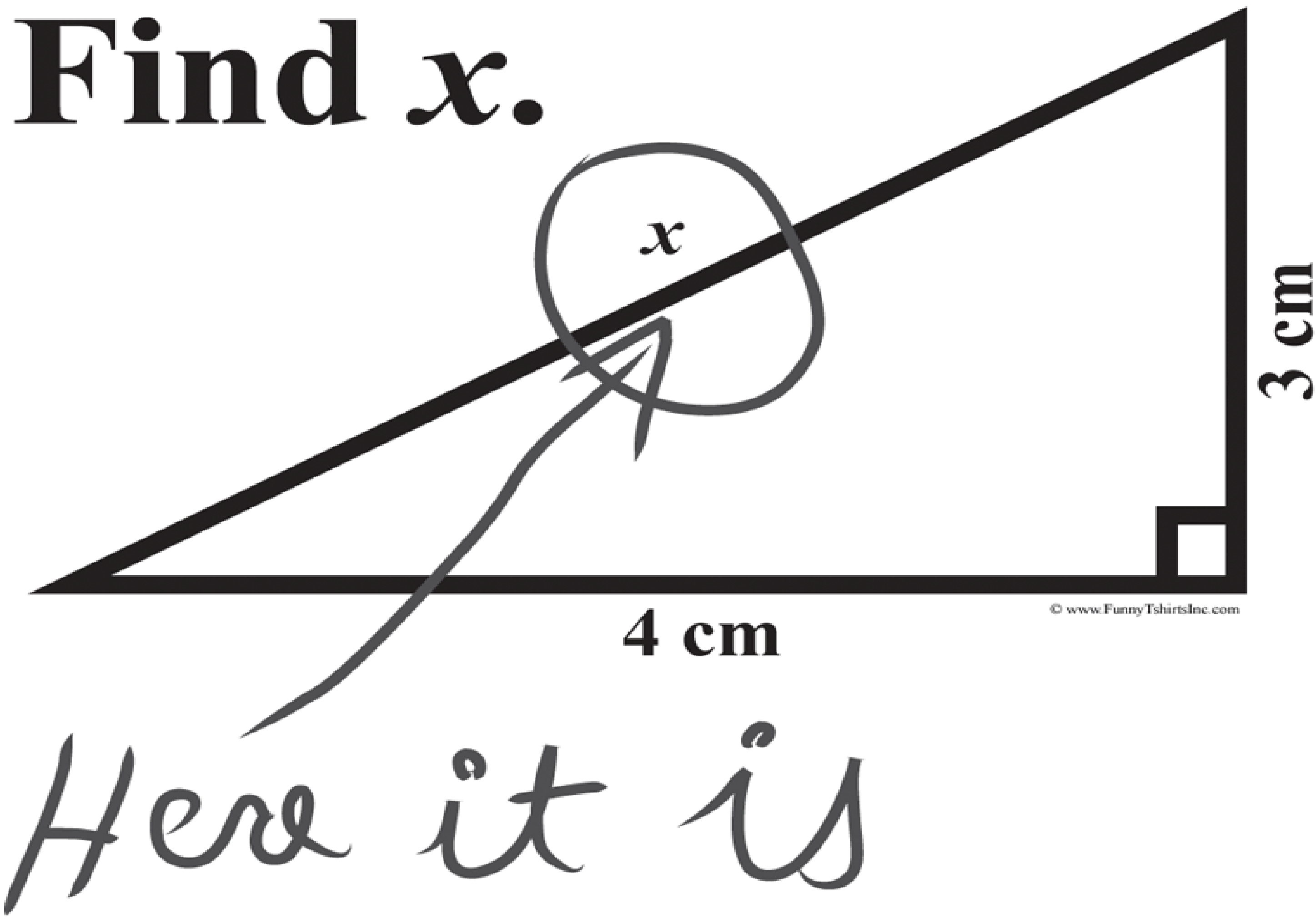
January 28th, 2012 Mock AMC 12 (H)

Welcome!

"There are only three kinds of people in the world. Those who can count, and those who can't."
-Oscar Wilde



Have Fun!

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MOCK 12

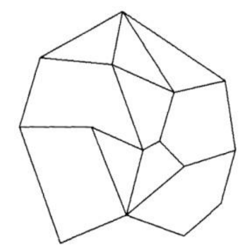
1. The difference between a positive fraction and its reciprocal is $\frac{9}{20}$. The sum of the fraction and its reciprocal is

(A) $2\frac{2}{9}$ (B) $2\frac{1}{20}$ (C) $2\frac{9}{20}$ (D) $4\frac{1}{20}$ (E) Not uniquely determined

2. If the larger root of the equation $(2011x)^2 - (2010 \times 2012)x - 1 = 0$ is m, and the smaller root of the equation $x^2 + 2010x - 2011 = 0$ is n, then m – n is

(A) 0 (B) 2012 (C) 2011 (D) $\frac{2011}{2012}$ (E) $\frac{2012}{2011}$

3. Isabella and Mark are guinea-pig farmers. They need to make enclosures for their many species of guinea-pigs, but unfortunately, they live in a country where there is a "fence tax." Consequently, they can only afford to erect a maximum of 24 fences. The enclosures can have any number of sides and be any shape, so long as the fences are straight, and a fence can join another fence only at end points. For example, see the illustration to the right. What is the largest number of enclosures they can make?



(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

4. The number of digits in the decimal expansion of 2^{2012} is closest to
   (A) 400 (B) 500 (C) 600 D) 700 (E) 800

5. Ten minutes before I put a cake in the oven, I put my cat outside. The cake had to cook for 35 minutes so I set the oven timer for 35 minutes. I immediately made myself a cup of coffee, which took me six minutes. Three minutes before I finished drinking my coffee, the cat came back inside. This was five minutes before the oven timer went off. The telephone rang halfway between the time I finished making my coffee and when the cat came inside. After a five minute conversation, I hung up. It was then 3:59 PM. How many minutes had the cat been outside when the telephone rang?

   (A) 25 (B) 26 C) 27 (D) 28 (E) 29

6. An urn contains 13 red marbles and 17 white marbles. One marble is randomly selected and its color noted. It is then returned to the urn along with 6 other marbles of the same color. A second marble is chosen at random. The probability that the second marble is red is

   (A) $\frac{13}{30}$ (B) $\frac{17}{30}$ (C) $\frac{13}{36}$ (D) $\frac{19}{36}$ (E) $\frac{5}{6}$

7. For how many values of x between 0.01 and 1 does the graph of the function $\sin(\frac{1}{x})$ cross the x-axis?

(A) 31 (B) 28 (C) 56 (D) 14 (E) 112

8. How many real solutions are there to the equation

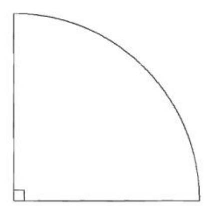
$$\sqrt[3]{2x+14} - \sqrt[3]{2x-14} = \sqrt[3]{4}$$

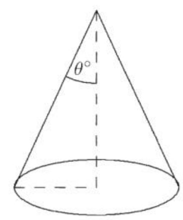
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

9. The notation $\lfloor x \rfloor$ means the largest integer not greater than x. The number of integers x lying between 0 and 500 for which $x - \lfloor \sqrt{x} \rfloor^2 = 10$ is

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

10. A quarter circle is folded to form a cone.

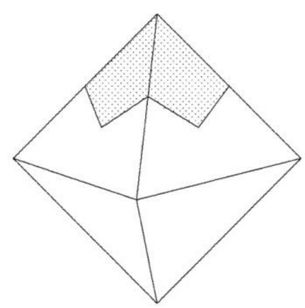




If θ is the angle between the vertical axis of symmetry of the cone and the slant height of the cone, then sin(θ) equals

(A) $\frac{1}{4}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{\sqrt{3}}{3}$

11. A regular octahedron has eight triangular faces and all sides the same length and has volume 120 inches^3. A solid portion of the regular octahedron consists of that part of it which is closer to the top vertex than to any other one. In the diagram, the outside part of this portion is shown shaded, and it extends into the octahedron.



What is the volume, in cubic inches, of this unusually shaped portion?

(A) 10 (B) $12\sqrt{2}$ (C) $8(\sqrt{2}+1)$ (D) $15\sqrt{2}$ (E) 20

12. The number 2000 = $2^4 \times 5^3$ is the product of seven prime factors. Let x be the smallest integer greater than 2000 with this property and y be the largest integer smaller than 2000 with this property. What is the value of x – y?

(A) 100 (B) 64 (C) 280 (D) 203 (E) 96

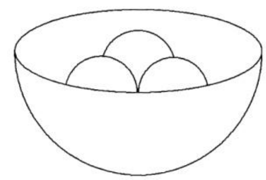
13. Bergen Academies has decided to create a pack of Math Team cards. Each card has the photo of a math team member with a unique natural number on the back of the card. Yuriko takes 4 of these cards and lays them out on a table. James selects 3 of the 4, looks at the numbers and says they add to 14. Jenny also selects 3 of the same four cards, looks at the numbers and says they add to 18. Roger similarly looks at the backs of three of the same four cards and says they add to 19. How many combinations of numbers could have been represented by the four cards?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

14. Given that $f(8) = 8 \ and \ f(n+3) = \frac{f(n)-1}{f(n)+1}$ for all n, then the value of f(2012) is

(A) 8 (B) $-\frac{1}{8}$ (C) 2012 (D) $-\frac{1}{2012}$ (E) $\frac{1}{64}$

15. When three spherical balls, each of radius 10 inches, are placed in a hemispherical dish, it is notice that the tops of the balls are all exactly level with the top of the dish. What is the radius, in inches, of the dish?



(A) $10(2\sqrt{\frac{7}{3}} - \frac{1}{2})$ (B) $10\ (\sqrt{\frac{7}{3}} + 1)$ (C) $10\ (\sqrt{3} + 1)$

(D) $10\ (3\sqrt{\frac{7}{3}} - 2)$ (E) 25

16. The function $y = f(x)$ is a function such that $f(f(x)) = 504x - 2012$ for every real number x. An integer t satisfies the equation $f(t) = 504t - 2012$. What is the value of t?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 503

17. The number of real solutions to the equation $4x^2 - 40\lfloor x \rfloor + 51 = 0$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

18. The following equation is x has two solutions r and s.

$$2\log_x 16 - \log_{16x} 16 - 3\log_{256x} 16 = 0$$

What is the value of $\log_2 rs$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. The smallest possible value that the expression

$$\sqrt{x^2 + y^2} + \sqrt{(x-1)^2 + y^2} + \sqrt{x^2 + (y-1)^2} + \sqrt{(x-3)^2 + (y-4)^2}$$

can have is

(A) 5 (B) $4 + \sqrt{3}$ (C) 6 (D) $5 + \sqrt{2}$ (E) 7

20. A sequence of number a_1, a_2, a_3 , … Is defined by $a_1 = 1$ and then after that, each new term is found by the relation $a_{n+1} = a_n(a_n + 2)$. For example, the next term in the sequence is $a_2 = 1 \times 3 = 3$. Which is the first term which exceeds 1000000000 ?

(A) a_5 (B) a_6 (C) a_7 (D) a_{10} (E) a_{30}

21. Let T_n be the right triangle with sides of length $(4n^2, 4n^4\text{-}1, 4n^4\text{+}1)$, n a positive integer, and let α_n be the angle opposite the side of length $4n^2$. Let $S = \alpha_1 + \alpha_2 + \alpha_3 + \dots$. Then S =

(A) $\frac{\pi}{4}$ (B) $\frac{4\pi}{7}$ (C) $\frac{\pi}{2}$ (D) π (E) ∞

22. In ΔABC, with sides a, b, and c, the angles have measure $\angle A = 54°$, $\angle B = 48°$, and $\angle C = 78°$. Side a is opposite $\angle A$ a, side b is opposite $\angle B$, and side C is opposite $\angle C$. For what value of k, does $c^2 = ka^2 + b^2$?

(A) $\frac{2\sqrt{5}+1}{8}$ (B) $\frac{3\sqrt{5}-4}{4}$ (C) $\frac{\sqrt{5}+1}{4}$ (D) $\frac{\sqrt{5}-1}{2}$ (E) $\frac{3}{5}$

23. How many pairs of positive integers have greatest common divisor 3! And least common multiple 18!

(A) 64 (B) 128 (C) 256 D) 435 (E) 562

24. Find the remainder when $A = 3^3 \times 33^{33} \times 333^{333} \times 3333^{3333}$ is divided by 100.

(A) 11 (B) 13 (C) 17 (D) 19 (E) 23

25. During a movie shoot, a stuntman jumps out of a plane and parachutes to safety within a 100 foot by 100 foot square field, which is entirely surrounded by a wooden fence. There is a flagpole in the middle of the square field. Assuming the stuntman is equally likely to land on any point in the field, the probability that he lands closer to the fence than to the flagpole can be written in simplest terms as $\frac{a-b\sqrt{c}}{d}$ where all four variables are positive integers, c is a multiple of no perfect square greater than 1, a is relatively prime to d, and b is relatively prime with d. Find the value of a + b + c + d.

(A) 11 (B) 13 (C) 17 (D) 19 (E) 23

26. (Extra Credit) A block Z is formed by gluing one face of a solid cube with side length 6 onto one of the circular faces of a right circular cylinder with radius 10 and height 3 so that the centers of the square side of the cube and circular top of the center coincide. If V is the smallest convex region that contains Z, calculate $\lfloor vol\ V \rfloor$ (the greatest integer less than or equal to the volume of V).

(A) 1881 (B) 1882 (C) 1883 (D) 1884 (E) 1885