2015

# AMC 12 B

## DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 25, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 25, 2015. Nothing is needed from inside this package until February 25.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*


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**MAA American Mathematics Competitions**

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MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions

66th Annual

# AMC 12 B

**American Mathematics Contest 12 B**

**Wednesday, February 25, 2015**

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

*Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 33rd annual American Invitational Mathematics Examination (AIME) on Thursday, March 19, 2015 or Wednesday, March 25, 2015. More details about the AIME are on the back page of this test booklet.*

1. What is the value of $2-(-2)^{-2}$?

   **(A)** $-2$ **(B)** $\frac{1}{16}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{9}{4}$ **(E)** 6

2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

   **(A)** 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

3. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

   **(A)** 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

4. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

   **(A)** David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

5. The Tigers beat the Sharks 2 out of the first 3 times they played. They then played $N$ more times, and the Sharks ended up winning at least 95% of all the games played. What is the minimum possible value for $N$?

   **(A)** 35 **(B)** 37 **(C)** 39 **(D)** 41 **(E)** 43

6. Back in 1930, Tillie had to memorize her multiplication facts from $0\times 0$ through $12\times 12$. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

   **(A)** 0.21 **(B)** 0.25 **(C)** 0.46 **(D)** 0.50 **(E)** 0.75

7. A regular 15-gon has $L$ lines of symmetry, and the smallest positive angle for which it has rotational symmetry is $R$ degrees. What is $L+R$?

   **(A)** 24 **(B)** 27 **(C)** 32 **(D)** 39 **(E)** 54

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## American Mathematics Competitions

### WRITE TO US!

*Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:*

MAA American Mathematics Competitions
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Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

*The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:*

Jerrold W. Grossman

### 2015 AIME

The 33rd annual AIME will be held on Thursday, March 19, with the alternate on Wednesday, March 25. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 44th Annual USA Mathematical Olympiad (USAMO) on April 28–29, 2015. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

### PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

25. A bee starts flying from point $P_0$. She flies 1 inch due east to point $P_1$. For $j \geq 1$, once the bee reaches point $P_j$, she turns $30^\circ$ counterclockwise and then flies $j+1$ inches straight to point $P_{j+1}$. When the bee reaches $P_{2015}$ she is exactly $a\sqrt{b} + c\sqrt{d}$ inches away from $P_0$, where $a$, $b$, $c$, and $d$ are positive integers and $b$ and $d$ are not divisible by the square of any prime. What is $a+b+c+d$?

**(A)** 2016 **(B)** 2024 **(C)** 2032 **(D)** 2040 **(E)** 2048



8. What is the value of $\left(625^{\log_5 2015}\right)^{\frac{1}{4}}$?

**(A)** 5 **(B)** $\sqrt[4]{2015}$ **(C)** 625 **(D)** 2015 **(E)** $\sqrt[4]{5^{2015}}$

9. Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\frac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

**(A)** $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

10. How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

11. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

**(A)** 20 **(B)** $\frac{360}{17}$ **(C)** $\frac{107}{5}$ **(D)** $\frac{43}{2}$ **(E)** $\frac{281}{13}$

12. Let $a$, $b$, and $c$ be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b) + (x-b)(x-c) = 0$?

**(A)** 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

13. Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70^\circ$, $\angle ADB = 40^\circ$, $AD = 4$, and $BC = 6$. What is $AC$?

**(A)** $3 + \sqrt{5}$ **(B)** 6 **(C)** $\frac{9}{2}\sqrt{2}$ **(D)** $8 - \sqrt{2}$ **(E)** 7

14. A circle of radius 2 is centered at $A$. An equilateral triangle with side 4 has a vertex at $A$. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

**(A)** $8 - \pi$ **(B)** $\pi + 2$ **(C)** $2\pi - \frac{\sqrt{2}}{2}$ **(D)** $4(\pi - \sqrt{3})$ **(E)** $2\pi + \frac{\sqrt{3}}{2}$

15. At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5?

**(A)** $\dfrac{11}{72}$ **(B)** $\dfrac{1}{6}$ **(C)** $\dfrac{3}{16}$ **(D)** $\dfrac{11}{24}$ **(E)** $\dfrac{1}{2}$

16. A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

**(A)** 18 **(B)** 162 **(C)** $36\sqrt{21}$ **(D)** $18\sqrt{138}$ **(E)** $54\sqrt{21}$

17. An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed $n$ times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of $n$?

**(A)** 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

18. For every composite positive integer $n$, define $r(n)$ to be the sum of the factors in the prime factorization of $n$. For example, $r(50) = 12$ because the prime factorization of 50 is $2 \cdot 5^2$, and $2 + 5 + 5 = 12$. What is the range of the function $r$, $\{r(n) : n \text{ is a composite positive integer}\}$?

**(A)** the set of positive integers

**(B)** the set of composite positive integers

**(C)** the set of even positive integers

**(D)** the set of integers greater than 3

**(E)** the set of integers greater than 4

19. In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points $X$, $Y$, $Z$, and $W$ lie on a circle. What is the perimeter of the triangle?

**(A)** $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32



20. For every positive integer $n$, let $\text{mod}_5(n)$ be the remainder obtained when $n$ is divided by 5. Define a function $f : \{0, 1, 2, 3, \ldots\} \times \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3, 4\}$ recursively as follows:

$$f(i,j) = \begin{cases} \text{mod}_5(j+1) & \text{if } i = 0 \text{ and } 0 \le j \le 4, \\ f(i-1, 1) & \text{if } i \ge 1 \text{ and } j = 0, \text{ and} \\ f(i-1, f(i, j-1)) & \text{if } i \ge 1 \text{ and } 1 \le j \le 4. \end{cases}$$

What is $f(2015, 2)$?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let $s$ denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of $s$?

**(A)** 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

22. Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

**(A)** 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

23. A rectangular box measures $a \times b \times c$, where $a$, $b$, and $c$ are integers and $1 \le a \le b \le c$. The volume and the surface area of the box are numerically equal. How many ordered triples $(a, b, c)$ are possible?

**(A)** 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

24. Four circles, no two of which are congruent, have centers at $A$, $B$, $C$, and $D$, and points $P$ and $Q$ lie on all four circles. The radius of circle $A$ is $\frac{5}{8}$ times the radius of circle $B$, and the radius of circle $C$ is $\frac{5}{8}$ times the radius of circle $D$. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let $R$ be the midpoint of $\overline{PQ}$. What is $AR + BR + CR + DR$?

**(A)** 180 **(B)** 184 **(C)** 188 **(D)** 192 **(E)** 196