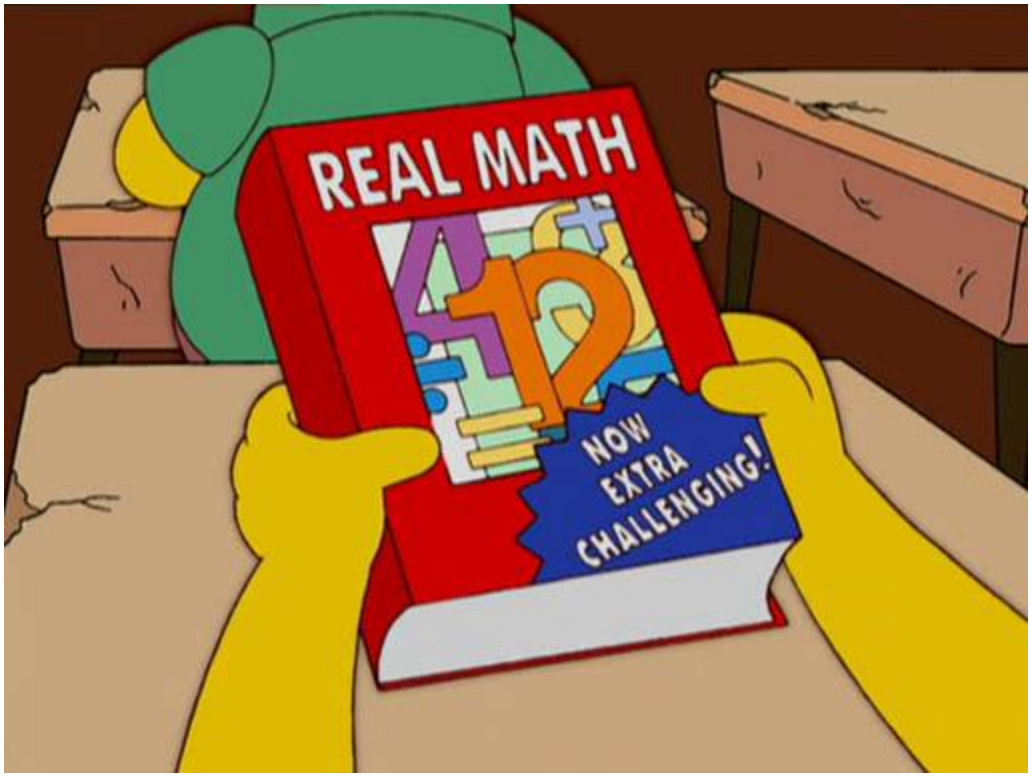
January 6th, 2018 Mock (Practice) AMC 10

Welcome!

$2018 = (10 + 987) \times (6 - 5) + 4 \times 3) \times 2 \times 1$

$2018 = 10 \times 9 + 8 \times 7 - 6 + 5^4 \times 3 + 2 + 1$



2016 = $2^5 \cdot 3^2 \cdot 7^1$
2017 = prime number
2018 = $2 \cdot 1009$
2019 = $3 \cdot 673$
2020 = $2 \cdot 2 \cdot 5 \cdot 101$
…
2027 = prime

2017 is a prime number. The next prime number after 2017 is 2027.

2018 is the sum of two squares $2018 = 43^2 + 13^2$

1009 is the sum of two squares $1009 = 15^2 + 28^2$

Daniel Plotnick

1. Given that A2018B is a 6 digit number divisible by 36. What is the largest such number?

a) 320184 b) 720180 c) 820188 d) 910188 e) 920188

2. Given that a, b, and c are positive even integers which satisfy the equation a + b + c = 2018. How many solutions does the equation have?

a) 403200 b) 507528 c) 508536 d) 510549 e) 2035153

3. Given a sequence such that $A_0 = 2$, $A_1 = -1$ and $A_{n+1} = 2A_n - A_{n-1}$, where $n \geq 1$, determine the value of A_{2018}.

a) 6049 b) --6049 c) 0 d) 6052 e) -6052

4. Given that a, b, c, and d are positive integers and ac+ bc+ ad + bd = 2018. Find the value of a+b+c+d.

a) 509 b) 1009 c) 1011 d) 2018 e) 2019

5. Suppose for real numbers $x_1, x_2, \ldots\ x_{2018}$ the following equation holds:

$$\sqrt{x_1 - 1} + \sqrt{x_2 - 1} + \cdots + \sqrt{x_{2018} - 1} = \frac{1}{2}(x_1 + x_2 + \cdots + x_{2018})$$

Find the value of $x_1 + x_2 + \cdots + x_{2018}$.

a) 2018 b) $1009\sqrt{5}$ c) $2018\sqrt{2}$ d) 3027 e) 4036

6. Given that a, b, and c are positive integers, and a + b = 2018, c – a = 2017, and a < b. Find the greatest possible value of a + b + c.

a) 4036 b) 5043 c) 5044 d) 5045 e) 6052

7. Find the value of the units digit of $1^2 + 2^2 + 3^2 + \cdots + 20182018^2$.

a) 1 b) 4 c) 5 d) 8 e) 9

8. Let p and q be the roots of the quadratic equation $x^2 - 14x + 1 = 0$

Find the value of $\frac{p^2}{q^2+1} + \frac{q^2}{p^2+1}$

a) 193 b) 195 c) $165\sqrt{3}$ d) $\frac{193\sqrt{3}}{2}$ e) 197

9. Let a_n and b_n be the x-intercepts of the quadratic equation

$$y = n(n-1)x^2 - (2n-1)x + 1$$

where n is an integer greater than 1. Find the value of

$$a_2b_2 + a_3b_3 + \ldots + a_{2018}b_{2018}$$

a) $\frac{2017}{2018}$ b) $\frac{2018}{2019}$ c) $\frac{2018}{2017}$ d) $\frac{2019}{2018}$ e) 1

10. Find the value of $\frac{1^4+2017^4+2018^4}{1^2+2017^2+2018^2}$

a) $2018^2 + 2018$ b) $2018^2 + 2017$ c) $2017^2 + 2017$ d) 2017^2+2018 e) none of these

11. We will call a number good if the binary representation of a positive integer has the following properties:
1) The number of digits is 11. 2) The number of 1s is 7 and number of 0s is 4.
For example, 2018 is a "good" number. 2018 = 11111100010
Find the sum of all "good" numbers.

a) 214936 b) 322455 c) 386946 d) 429876 e) 506716

12. Let A, B, and C be three digits. The number formed by these digits has the following properties:

i) ACB is divisible by 3 ii) BAC is divisible by 4
iii) BCA is divisible by 5 iv) CBA has an odd number of factors

Find the 3-digit number ABC.

a) 625 b) 582 c) 526 d) 522 e) 255

13. How many pairs of distinct integers between 1 and 2018 inclusive have their product as multiples of 5?

a) 650845 b) 731848 c) 812851 d) 813254 e) 1140574

14. Given that x is a real number, find the least value of $\sqrt{2x(x+5)(x+6)(x+11)+2018}$

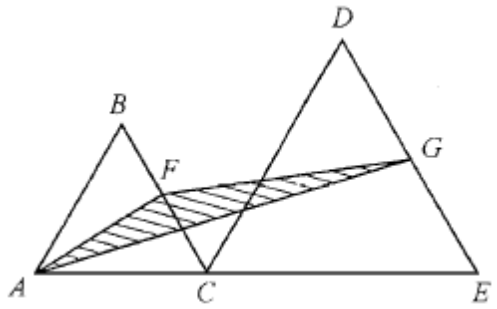
a) $22\sqrt{2}$ b) $24\sqrt{2}$ c) $26\sqrt{2}$ d) $28\sqrt{2}$ e) $30\sqrt{2}$

15. If x is real number, find the minimum value of the following function:

$f(x) = |x-1| + |x-2| + \ldots + |x-2018|$.

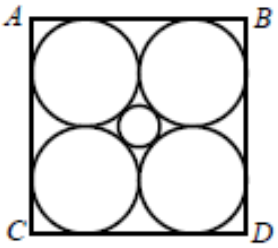
a) 1009^2 b) $\frac{2018^2}{2}$ c) $\frac{2018\times2019}{2}$ d) $\frac{2018\times2019}{4}$ e) $\frac{2018^2}{4} - 1$

16. In the figure at right, C lies on AE, ΔABC and ΔCDE are equilateral triangles, F and G are the mid-points of BC and DE respectively. If the area of ΔABC is 24 cm^2, the area of ΔCDE is 60 cm^2, and the area of ΔAFG is X cm^2, find the value of X.



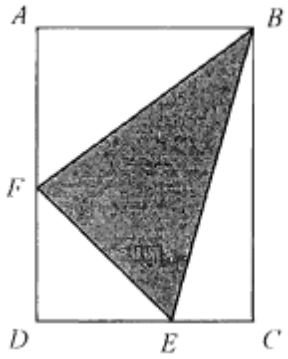
a) 12 b) 15 c) $12\sqrt{3}$ d) $8\sqrt{3}$ e) $\frac{25}{2}$

17. In the figure at right, $ABCD$ is a square of side length 2. Find the radius of the small circle at the center of the square.



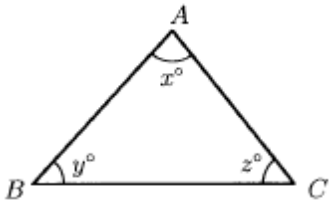
a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{2}-1}{2}$ c) $\frac{3\sqrt{2}-3}{2}$ d) $\frac{3\sqrt{2}-2}{2}$ e) $\frac{3\sqrt{2}-1}{4}$

18. In the figure at right, $ABCD$ is a rectangle. Points E and F lie on CD and AD respectively, such that AF = 8 cm and EC = 5 cm. Given that the area of the shaded region is 80 cm^2. Let the area of the rectangle $ABCD$ be X cm^2, find the value of X.



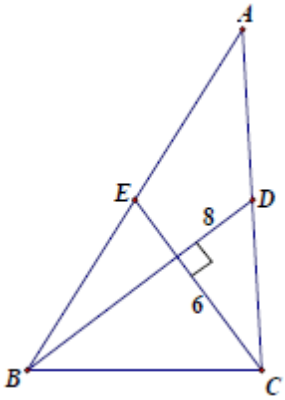
a) 154 b) 160 c) 180 d) 192 e) 200

19. In the figure at right, ABC is a triangle satisfying $x \geq y \geq z$ and $4x = 7z$. If the maximum value of x is m and the minimum value of x is n, find the value of $m + n$.



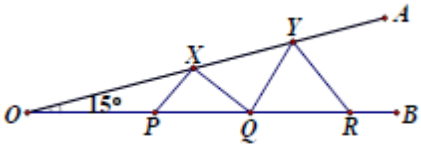
a) 146 b) 150 c) 154 d) 158 e) 162

20. In the figure at right BD and CE are the medians of the sides AC and AB of ΔABC respectively, and $BD \perp CE$. Given that BD = 8, CE = 6, find the area of ΔABC.



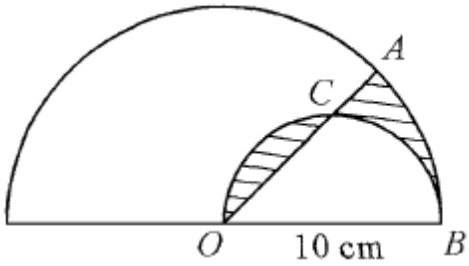
a) 72 b) 60 c) 48 d) 32 e) 24

21. In the figure at right, $\angle AOB = 15°$. X, Y are points on OA, P, Q, R are points on OB such that $OP = 3$ and $OR = 6$. If $s = PX + XQ + QY + YR$, find the least value of s.



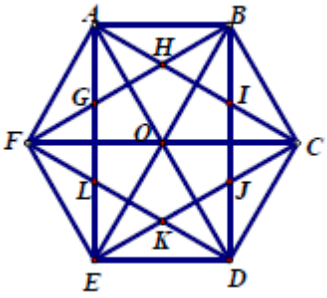
a) $\sqrt{12}$ b) 4 c) $3\sqrt{2}$ d) 5 e) $3\sqrt{3}$

22. In the figure at right, O is the center of the bigger semicircle with radius 10 cm, OB is the diameter of the smaller semicircle and C is the midpoint of arc OB and it lies on the segment OA. Let the area of the shaded region be K cm2, find the value of K.



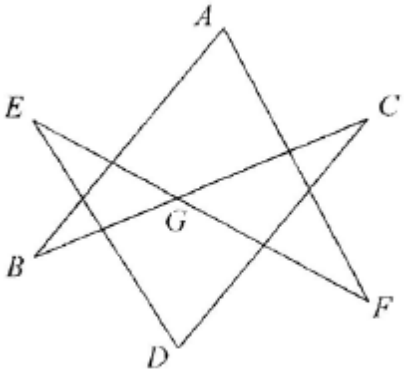
a) 8π b) 25 c) $\frac{25}{2}(\pi - 1)$ d) $\frac{25}{2}(\pi - 2)$ e) $\frac{25}{4}(2\pi - 1)$

23. In the figure at right, $ABCDEF$ is a regular hexagon and its area is $90\sqrt{3}$, find the length of GJ.



a) $4\sqrt{3}$ b) $4\sqrt{5}$ c) $6\sqrt{3}$ d) $6\sqrt{5}$ e) $9\sqrt{3}$

24. As shown in the figure at right, suppose $\angle EGB = 64°$, What is $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$?



a) 232 b) 236 c) 240 d) 244 e) 248

25. In the figure at right, X, Y and Z are points on BC, CA and AB of ΔABC respectively such that $\angle AZY = \angle BZX$, $\angle BXZ = \angle CXY$ and $\angle CYX = \angle AYZ$. If $AB = 10$, $BC = 6$ and $CA = 9$, find the length of AZ.

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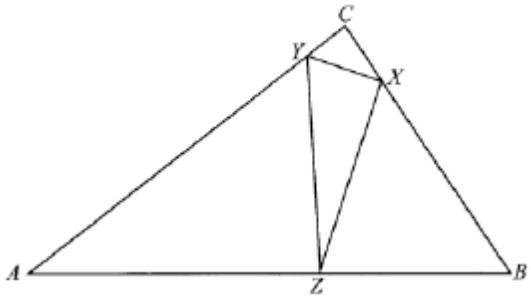

a) $\frac{25}{4}$ b) $\frac{26}{4}$ c) $\frac{27}{4}$ d)7 e) $\frac{29}{4}$