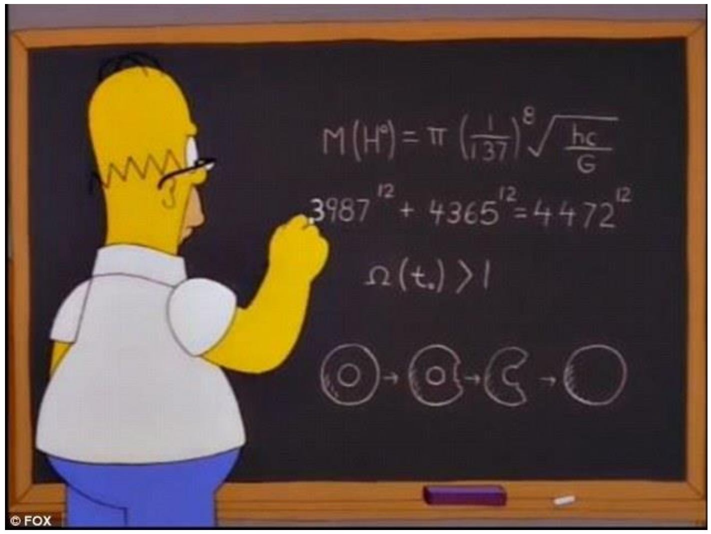
January 9th, 2016 Mock AMC <u>10</u>

Welcome!



2016 has 8 prime factors. $2016 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 * 3 * 7 = 2^5 3^2 7^1$.
2017 = prime
2016 = 2 · 2 · 2 · 2 · 2 · 3 · 3 · 7
2015 = 5 · 13 · 31
2014 = 2 · 19 · 53
2013 = 3 · 11 · 61
2012 = 2 · 2 · 503
2011 = prime

2016 is the 63rd triangular number, 1, 3, 6, 10, 15, …. . Triangular numbers are often denoted T_n. The recursive equation is $T_{n+1} = T_n + n$. The formula for T_n is $T_n = \frac{n(n+1)}{2}$.

If m is the nth triangular number then $m = \frac{\sqrt{8m+1}-1}{2}$.

This magic square is made up of only prime numbers. What is the magic number?

103	113	131	409	349	421	197	293
389	331	397	97	193	263	179	167
109	433	439	199	127	101	241	367
137	373	353	163	359	211	229	191
311	181	149	419	79	271	223	383
157	269	151	277	401	337	317	107
379	83	307	313	251	173	283	227
431	233	89	139	257	239	347	281

Daniel Plotnick

1. If $\frac{1}{2}$ $of\ a$ $\frac{1}{3}$ $of\ a$ $\frac{1}{4}$ $of\ a$ $\frac{1}{5}$ $of\ a$ $\frac{1}{6}$ $of\ a$ $\frac{1}{7}$ $of\ an\frac{1}{8}$ $of\ a$ $\frac{1}{9}$ $of\ 2016N\ is\ 1.$

What is the value of N?

A) 45 B) 90 C) 180 D) 360 E) 540

2. Let x be the difference between the least triangular number greater than 2016, and 2016. Let y be the difference between the least square number greater than 2016, and 2016. Let z be the difference between the least prime number greater than 2016, and 2016. Let w be the difference between the least cube number great than 2016, and 2016. Suppose

$$x + y + z + w = 2^N - 1, for\ some\ N.\text{ Then } N \text{ is}$$

A) 6 B) 7 C) 8 D) 9 E) 10

3. Let S be the sum of the first 2016 terms of the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, …. . Where the natural number n occurs as n terms. The sum of the digits of S is

A) 20 B) 24 C) 25 D) 30 E) 32

4. Let $a, b, c, and\ d$ be in arithmetic progression. $If\ a + d = 100, and\ bc = 99,$ then the value of the smallest of $a, b, c,\ and\ d$ is

A) 197 B) 97 C) 1 D) -97 E) -99

5. Adam ant is looking up towards the top of the flagpole in front of Bergen County Academies and notes that he is looking upward at an angle of 30 degrees. He then walks 20 ft. towards the flagpole and notices that now when he is looking up at the top of the flagpole he is looking up at an angle of 45 degrees. How tall is the flagpole?

A) $10 + 10\sqrt{3}$ B) $5 + 5\sqrt{3}$ C) $10 + 10\sqrt{2}$ D) $20 + 20\sqrt{3}$ E) 30

6. A really hard problem is given to three former BCA math team members, Kelvin, Ryan, and Alex. Their respective probabilities of solving the problem independently are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Calculate the probability that the problem gets solved?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{9}{10}$

7. A square with vertices A, B, C, and D is rotated 45◦ clockwise about its center, with the vertices now at A', B', C', and D' respectively. The eight line segments AA', A'B, BB', B'C, CC', C'D, DD', D'A are drawn forming an octagon. If the area of the square ABCD was 4, what is area of octagon AA'BB'CC'DD'?

A) $4\sqrt{2}$ + 2 B) $2\sqrt{2}$ + 4 C) $4\sqrt{2}$ D) 8 E) $4 + \sqrt{2}$

8. The number of pairs (a,b) of natural numbers, $a < b$, such that

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2016} \text{ is}$$

A) 81 B) 82 C) 164 D) 165 E) 330

9. Let S_1, S_2, … be squares such that for each n ≥ 1, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the area of S_1 is 100 in^2, then for which of the following values of n is the area of square S_n first less than 1 in^2.

A) 7 B) 8 C) 9 D) 10 E) 11

10. Vivian flips a fair coin a number of times and gets 2 points for each head she gets and 1 point for each tail. The probability that she gets exactly six points is

A) $\frac{11}{16}$ B) $\frac{42}{64}$ C) $\frac{41}{64}$ D) $\frac{5}{8}$ E) $\frac{43}{64}$

11. Let r and s be the roots of the equation $4^x - 3(2^{x+3}) + 128 = 0$, then the value of $\frac{r+s}{|r-s|}$ is

A) 3 B) 7 C) 12 D) 14 E) 128

12. If the letters of the word "ASSASSIN" are written down at random in a row, the probability that no two S's occur together is

A) $\frac{1}{7}$ B) $\frac{1}{8}$ C) $\frac{1}{10}$ D) $\frac{1}{14}$ E) $\frac{1}{20}$

13. If α and β are roots of $x^2 - 3x + 5 = 0$, and r, s are the roots of $x^2 + 5x - 3 = 0$,

and αr +βs, αs+βr are the roots of $x^2 + px + q = 0$, then p + q is

A) 143 B) 144 C) 173 D) 174 E) 177

14. The sides AB, BC, and CA of a triangle ABC have n, n+1, and n+2 (n a natural number n >= 3) interior points respectively on them. If the number of triangles formed by any three of these 3n+3 points is 421, then n is

A) 7 B) 6 C) 5 D) 4 E) 3

15. The sum of the real roots of the equation $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ is

A) -2 B) 2 C) 3 D) $2\sqrt{3}$ E) 4

16. Two numbers x and y are selected from the set of the first 25 natural numbers. The number of ways of selecting them such that $x^2 - y^2$ is divisible by 5 is

A) 200 B) 150 C) 125 D) 100 E) 75

17. What is the minimum possible perimeter of a triangle with one vertex at (3,9) and one vertex anywhere on the y-axis and one vertex anywhere on the line y = x?

A) $3(1+\sqrt{2}+\sqrt{3})$ B) $6\sqrt{5}$ C) $6+6\sqrt{2}$ D) 12 E) $6+3\sqrt{5}$

18. Suppose the ratio of the nth terms of two arithmetic progressions is $\frac{14n-6}{8n+23}$. What is the ratio of the first m terms of the arithmetic progressions?

A) $\frac{7m+1}{4m+27}$ B) $\frac{7m+1}{7m+24}$ C) $\frac{9m-1}{4m+27}$ D) $\frac{4m+4}{7m+24}$ E) $\frac{4m+4}{4m+27}$

19. The number of lattice points, having both coordinates integers, that lie in the interior of the triangle AOB with vertices O(0,0), A(0,41), and B(41,0) is

A) 780 B) 779 C) 903 D) 861 E) 820

20. In triangle ABC with sides a, b, and c, a = $3\sqrt{3}$, and $c = 2b$. The maximum area of triangle ABC is

A) 8 B) $5\sqrt{3}$ C) 9 D) $6\sqrt{3}$ E) 10

21. Find the value of the infinite product $\prod_{n=3}^{\infty}\frac{n^3-8}{n^3+8} = \left(\frac{3^3-8}{3^3+8}\right)\left(\frac{4^3-8}{4^3+8}\right)\left(\frac{5^3-8}{5^3+8}\right)\cdots$

A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{2}{5}$ E) $\frac{12}{19}$

22. In triangle ABC, A, B, and C are the vertices, and a, b, and c are the sides opposite vertices A, B, and C respectively. The incircle (inscribed circle) trisects the median from vertex A. If a = 10, then the inradius (the radius of the incircle) r is

A) $\frac{3\sqrt{14}}{7}$ B) $\frac{5\sqrt{7}}{7}$ C) $\frac{4\sqrt{14}}{7}$ D) $\frac{3\sqrt{7}}{7}$ E) $\frac{3\sqrt{10}}{7}$

23. Suppose $(10^{2016}+5)^2 = 225N$, then the sum of the digits of N is

A) 24,169 B) 24,189 C) 24,193 D) 24,181 E) 24,205

24. Let $\lfloor x \rfloor$ be the floor of x, that is, the greatest integer less than or equal to x, and let $\{\,x\,\} = x - \lfloor x \rfloor$, that is $\{\,x\,\}$ is the fractional part of x. Suppose $\{\,n^{-1}\} = \{\,n^2\}$, where n is positive and $2 < n^2 < 3$. Find the value of $n^{12} - 144n^{-1}$.

A) 279 B) 299 C) 266 D) 244 E) 233

25. Let $f\colon \mathbb{N} \to \mathbb{R}^+$ be a function from the natural numbers $\mathbb{N}$ to the positive real numbers $\mathbb{R}^+$. Such that $f(1) = 1$ and $f(1)+2f(2)+3f(3)+\cdots+nf(n) = n(n+1)f(n)$, for all n ∈ $\mathbb{N}$, n ≥ 2, where $\mathbb{N}$ is the set of natural numbers and R is the set of real numbers. Then the value of $f(2016)$ is

A) 4032 B) 2016 C) 1 D) $\frac{1}{2016}$ E) $\frac{1}{4032}$