2017

AMC 10B

DO NOT OPEN UNTIL WEDNESDAY, February 15, 2017

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the Teachers' Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 15, 2017.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found on amc.maa.org under 'AMC 10/12') that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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MAA
MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

18th Annual

AMC 10B

American Mathematics Competition 10B
Wednesday, February 15, 2017

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 35th annual American Invitational Mathematics Examination (AIME) on Thursday, March 7, 2017 or Wednesday, March 22, 2017. More details about the AIME are on the back page of this test booklet.

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1. Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

2. Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps?

(A) 5 minutes and 35 seconds **(B)** 6 minutes and 40 seconds
(C) 7 minutes and 5 seconds **(D)** 7 minutes and 25 seconds
(E) 8 minutes and 10 seconds

3. Real numbers x, y, and z satisfy the inequalities

$$0 < x < 1, \qquad -1 < y < 0, \qquad \text{and} \qquad 1 < z < 2.$$

Which of the following numbers is necessarily positive?

(A) $y + x^2$ **(B)** $y + xz$ **(C)** $y + y^2$ **(D)** $y + 2y^2$
(E) $y + z$

4. Suppose that x and y are nonzero real numbers such that

$$\frac{3x + y}{x - 3y} = -2.$$

What is the value of

$$\frac{x + 3y}{3x - y}\,?$$

(A) -3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** 3

5. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701
Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2017 AIME

The 35th annual AIME will be held on Thursday, March 7, 2017 with the alternate on Wednesday, March 22, 2017. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 46th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

23. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

(A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

24. The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

(A) 48 **(B)** 60 **(C)** 108 **(D)** 120 **(E)** 169

25. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

(A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

6. What is the largest number of solid 2-in $\times$ 2-in $\times$ 1-in blocks that can fit in a 3-in $\times$ 2-in $\times$ 3-in box?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

7. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

(A) 2.0 **(B)** 2.2 **(C)** 2.8 **(D)** 3.4 **(E)** 4.4

8. Points $A(11, 9)$ and $B(2, -3)$ are vertices of $\triangle ABC$ with $AB = AC$. The altitude from A meets the opposite side at $D(-1, 3)$. What are the coordinates of point C?

(A) $(-8, 9)$ **(B)** $(-4, 8)$ **(C)** $(-4, 9)$ **(D)** $(-2, 3)$
(E) $(-1, 0)$

9. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

(A) $\frac{1}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{7}{27}$ **(E)** $\frac{1}{2}$

10. The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is c?

(A) -13 **(B)** -8 **(C)** 2 **(D)** 8 **(E)** 13

11. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

(A) 10% **(B)** 12% **(C)** 20% **(D)** 25% **(E)** $33\frac{1}{3}\%$

12. Elmer's new car gets 50% better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car uses. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

    (A) 20% **(B)** $26\frac{2}{3}\%$ **(C)** $27\frac{7}{9}\%$ **(D)** $33\frac{1}{3}\%$ **(E)** $41\frac{2}{3}\%$

13. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. An integer N is selected at random in the range $1 \le N \le 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

    (A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{5}$ **(E)** 1

15. Rectangle $ABCD$ has $AB = 3$ and $BC = 4$. Point E is the foot of the perpendicular from B to diagonal $\overline{AC}$. What is the area of $\triangle ADE$?

    (A) 1 **(B)** $\frac{42}{25}$ **(C)** $\frac{28}{15}$ **(D)** 2 **(E)** $\frac{54}{25}$

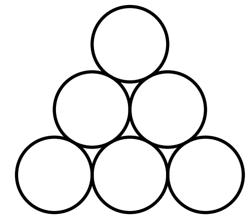
16. How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

    (A) 469 **(B)** 471 **(C)** 475 **(D)** 478 **(E)** 481

17. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

    (A) 1024 **(B)** 1524 **(C)** 1533 **(D)** 1536 **(E)** 2048

18. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



    (A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

19. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

    (A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$

20. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

    (A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

21. In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and D is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$?

    (A) $\sqrt{5}$ **(B)** $\frac{11}{4}$ **(C)** $2\sqrt{2}$ **(D)** $\frac{17}{6}$ **(E)** 3

22. The diameter $\overline{AB}$ of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment $\overline{AE}$ intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

    (A) $\frac{120}{37}$ **(B)** $\frac{140}{39}$ **(C)** $\frac{145}{39}$ **(D)** $\frac{140}{37}$ **(E)** $\frac{120}{31}$