2017

AMC 10A

DO NOT OPEN UNTIL TUESDAY, February 7, 2017

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the Teachers' Manual. PLEASE READ THE MANUAL BEFORE FEBRUARY 7, 2017.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found on amc.maa.org under 'AMC 10/12') that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions for this contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet, or media of any type is a violation of the competition rules.*

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MAA
MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

18th Annual

AMC 10A

American Mathematics Competition 10A

Tuesday, February 7, 2017

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the 35th annual American Invitational Mathematics Examination (AIME) on Thursday, March 7, 2017 or Wednesday, March 22, 2017. More details about the AIME are on the back page of this test booklet.

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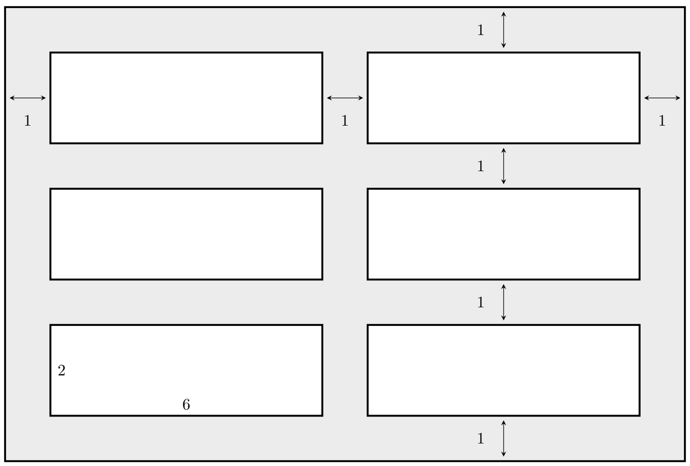
1. What is the value of $(2(2(2(2(2(2+1)+1)+1)+1)+1)+1)$?

   (A) 70 **(B)** 97 **(C)** 127 **(D)** 159 **(E)** 729

2. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

   (A) 8 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

3. Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?



   (A) 72 **(B)** 78 **(C)** 90 **(D)** 120 **(E)** 150

4. Mia is "helping" her mom pick up 30 toys that are strewn on the floor. Mia's mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

   (A) 13.5 **(B)** 14 **(C)** 14.5 **(D)** 15 **(E)** 15.5

5. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

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American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by MAA's Subcommittee on the AMC10/AMC12 Exams, under the direction of the co-chairs Jerrold W. Grossman and Carl Yerger.

2017 AIME

The 35th annual AIME will be held on Thursday, March 7, 2017 with the alternate on Wednesday, March 22, 2017. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you achieve a high score on this competition. Top-scoring students on the AMC 10/12/AIME will be selected to take the 46th Annual USA Mathematical Olympiad (USAMO) on April 19–20, 2017. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.maa.org/amc

21. A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

(A) $\dfrac{12}{13}$ **(B)** $\dfrac{35}{37}$ **(C)** 1 **(D)** $\dfrac{37}{35}$ **(E)** $\dfrac{13}{12}$

22. Sides $\overline{AB}$ and $\overline{AC}$ of equilateral triangle ABC are tangent to a circle at points B and C, respectively. What fraction of the area of $\triangle ABC$ lies outside the circle?

(A) $\dfrac{4\sqrt{3}\pi}{27}-\dfrac{1}{3}$ **(B)** $\dfrac{\sqrt{3}}{2}-\dfrac{\pi}{8}$ **(C)** $\dfrac{1}{2}$ **(D)** $\sqrt{3}-\dfrac{2\sqrt{3}\pi}{9}$ **(E)** $\dfrac{4}{3}-\dfrac{4\sqrt{3}\pi}{27}$

23. How many triangles with positive area have all their vertices at points (i,j) in the coordinate plane, where i and j are integers between 1 and 5, inclusive?

(A) 2128 **(B)** 2148 **(C)** 2160 **(D)** 2200 **(E)** 2300

24. For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

(A) -9009 **(B)** -8008 **(C)** -7007 **(D)** -6006 **(E)** -5005

25. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

(A) 226 **(B)** 243 **(C)** 270 **(D)** 469 **(E)** 486

6. Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

(A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong.

(B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.

(C) If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.

(D) If Lewis received an A, then he got all of the multiple choice questions right.

(E) If Lewis received an A, then he got at least one of the multiple choice questions right.

7. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

(A) 30% **(B)** 40% **(C)** 50% **(D)** 60% **(E)** 70%

8. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

(A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

9. Minnie rides on a flat road at 20 kilometers per hour (kph), downhill at 30 kph, and uphill at 5 kph. Penny rides on a flat road at 30 kph, downhill at 40 kph, and uphill at 10 kph. Minnie goes from town A to town B, a distance of 10 km all uphill, then from town B to town C, a distance of 15 km all downhill, and then back to town A, a distance of 20 km on the flat. Penny goes the other way around using the same route. How many more minutes does it take Minnie to complete the 45-km ride than it takes Penny?

(A) 45 **(B)** 60 **(C)** 65 **(D)** 90 **(E)** 95

10. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

11. The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

(A) 6 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

12. Let S be the set of points (x, y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description of S?

(A) a single point **(B)** two intersecting lines

(C) three lines whose pairwise intersections are three distinct points

(D) a triangle **(E)** three rays with a common endpoint

13. Define a sequence recursively by $F_0 = 0$, $F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$. What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda?

(A) 9% **(B)** 19% **(C)** 22% **(D)** 23% **(E)** 25%

15. Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloé's number?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

16. There are 10 horses, named Horse 1, Horse 2, $\ldots$, Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

17. Distinct points P, Q, R, and S lie on the circle $x^2 + y^2 = 25$ and have integer coordinates. The distances PQ and RS are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

(A) 3 **(B)** 5 **(C)** $3\sqrt{5}$ **(D)** 7 **(E)** $5\sqrt{2}$

18. Amelia has a coin that lands on heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. The probability that Amelia wins is $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $q - p$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

(A) 12 **(B)** 16 **(C)** 28 **(D)** 32 **(E)** 40

20. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

(A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265