


2015
AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, February 25, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 25, 2015. Nothing is needed from inside this package until February 25.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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MAA100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

American Mathematics Competitions
16th Annual
AMC 10 B
American Mathematics Contest 10 B
Wednesday February 25, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 33rd annual American Invitational Mathematics Examination (AIME) on Thursday, March 19, 2015 or Wednesday, March 25, 2015. More details about the AIME are on the back page of this test booklet.

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1. What is the value of $2 - (-2)^{-2}$?

(A) -2 (B) $\frac{1}{16}$ (C) $\frac{7}{4}$ (D) $\frac{9}{4}$ (E) 6

2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

(A) 3:10 PM (B) 3:30 PM (C) 4:00 PM (D) 4:10 PM (E) 4:30 PM

3. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

(A) 8 (B) 11 (C) 14 (D) 15 (E) 18

4. Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of the pizza they consumed?

(A) Alex, Beth, Cyril, Dan
 (B) Beth, Cyril, Alex, Dan
 (C) Beth, Cyril, Dan, Alex
 (D) Beth, Dan, Cyril, Alex
 (E) Dan, Beth, Cyril, Alex

5. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

(A) David (B) Hikmet (C) Jack (D) Rand (E) Todd

6. Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

(A) Sunday (B) Tuesday (C) Thursday (D) Friday (E) Saturday



American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

MAA American Mathematics Competitions
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 Annapolis Junction, MD 20701
 Phone 800.527.3690 | Fax 240.396.5647 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Silvia Fernandez

2015 AIME

The 33rd annual AIME will be held on Thursday, March 19, with the alternate on Wednesday, March 25. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 44th Annual USA Mathematical Olympiad (USAMO) on April 28–29, 2015. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

23. Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let s denote the sum of the four least possible values of n . What is the sum of the digits of s ?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

24. Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0 = (0, 0)$ facing to the east and walks one unit, arriving at $p_1 = (1, 0)$. For $n = 1, 2, 3, \dots$, right after arriving at the point p_n , if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1} , he does that. Otherwise, he walks one unit straight ahead to reach p_{n+1} . Thus the sequence of points continues $p_2 = (1, 1)$, $p_3 = (0, 1)$, $p_4 = (-1, 1)$, $p_5 = (-1, 0)$, and so on in a counterclockwise spiral pattern. What is p_{2015} ?

(A) $(-22, -13)$ (B) $(-13, -22)$ (C) $(-13, 22)$ (D) $(13, -22)$

(E) $(22, -13)$

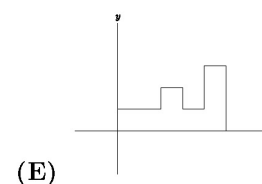
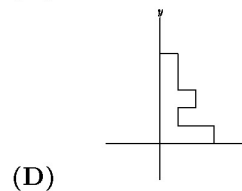
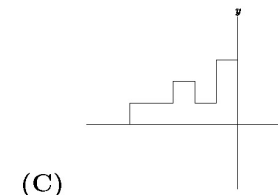
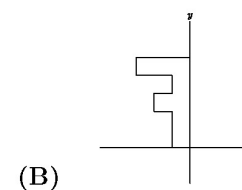
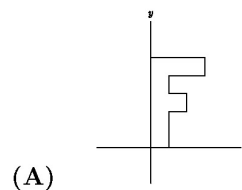
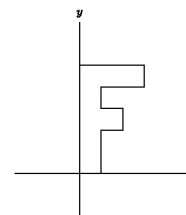
25. A rectangular box measures $a \times b \times c$, where a , b , and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

(A) 4 (B) 10 (C) 12 (D) 21 (E) 26

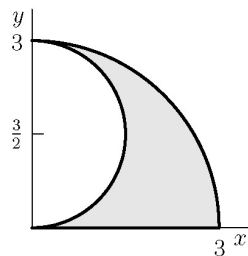
7. Consider the operation “minus the reciprocal of,” defined by $a \diamond b = a - \frac{1}{b}$. What is $((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3))$?

(A) $-\frac{7}{30}$ (B) $-\frac{1}{6}$ (C) 0 (D) $\frac{1}{6}$ (E) $\frac{7}{30}$

8. The letter F shown below is rotated 90° clockwise around the origin, then reflected in the y -axis, and then rotated a half turn around the origin. What is the final image?



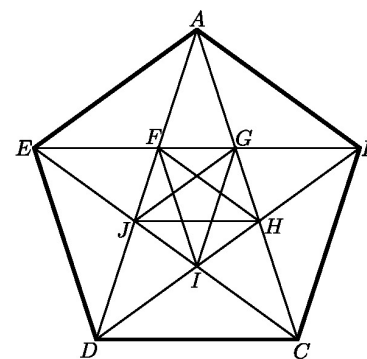
9. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0, 0)$ that lies in the first quadrant, the portion of the circle of radius $\frac{3}{2}$ and center $(0, \frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0, 0)$ to $(3, 0)$. What is the area of the shark's fin falcata?



- (A) $\frac{4\pi}{5}$ (B) $\frac{9\pi}{8}$ (C) $\frac{4\pi}{3}$ (D) $\frac{7\pi}{5}$ (E) $\frac{3\pi}{2}$
10. What are the sign and units digit of the product of all the odd negative integers strictly greater than -2015 ?
- (A) It is a negative number ending with a 1.
 (B) It is a positive number ending with a 1.
 (C) It is a negative number ending with a 5.
 (D) It is a positive number ending with a 5.
 (E) It is a negative number ending with a 0.
11. Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?
- (A) $\frac{8}{99}$ (B) $\frac{2}{5}$ (C) $\frac{9}{20}$ (D) $\frac{1}{2}$ (E) $\frac{9}{16}$
12. For how many integers x is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?
- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15
13. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

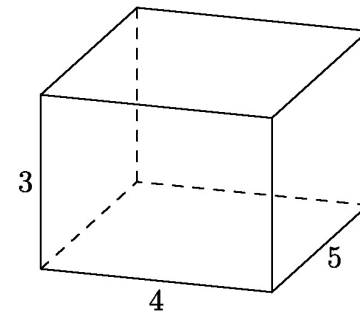
- (A) 20 (B) $\frac{360}{17}$ (C) $\frac{107}{5}$ (D) $\frac{43}{2}$ (E) $\frac{281}{13}$

19. In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z , and W lie on a circle. What is the perimeter of the triangle?
- (A) $12 + 9\sqrt{3}$ (B) $18 + 6\sqrt{3}$ (C) $12 + 12\sqrt{2}$ (D) 30 (E) 32
20. Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?
- (A) 6 (B) 9 (C) 12 (D) 18 (E) 24
21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s ?
- (A) 9 (B) 11 (C) 12 (D) 13 (E) 15
22. In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?



- (A) 3 (B) $12 - 4\sqrt{5}$ (C) $\frac{5 + 2\sqrt{5}}{3}$ (D) $1 + \sqrt{5}$ (E) $\frac{11 + 11\sqrt{5}}{10}$

14. Let a , b , and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?
- (A) 15 (B) 15.5 (C) 16 (D) 16.5 (E) 17
15. The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?
- (A) 41 (B) 47 (C) 59 (D) 61 (E) 66
16. Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?
- (A) $\frac{9}{1000}$ (B) $\frac{1}{90}$ (C) $\frac{1}{80}$ (D) $\frac{1}{72}$ (E) $\frac{2}{121}$
17. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of the octahedron?



- (A) $\frac{75}{12}$ (B) 10 (C) 12 (D) $10\sqrt{2}$ (E) 15
18. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?
- (A) 32 (B) 40 (C) 48 (D) 56 (E) 64