2015

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, February 3, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 3, 2015. Nothing is needed from inside this package until February 3.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

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MAA
MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

16th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 3, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 33rd annual American Invitational Mathematics Examination (AIME) on Thursday, March 19, 2015 or Wednesday, March 25, 2015. More details about the AIME and other information are on the back page of this test booklet.

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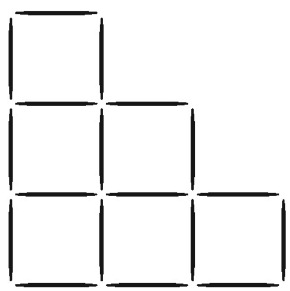
1. What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

   (A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2. A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

   (A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

3. Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does she need to add to complete a 5-step staircase?



   (A) 9 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

4. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

   (A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

5. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

   (A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

<u>WRITE TO US!</u>

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Silvia Fernandez

<u>2015 AIME</u>

The 33rd annual AIME will be held on Thursday, March 19, with the alternate on Wednesday, March 25. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 44th Annual USA Mathematical Olympiad (USAMO) on April 28–29, 2015. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

24. For some positive integers p, quadrilateral $ABCD$ with positive integer side lengths has perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

25. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

6. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

(A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

7. How many terms are there in the arithmetic sequence $13, 16, 19, \dots, 70, 73$?

(A) 20 **(B)** 21 **(C)** 24 **(D)** 60 **(E)** 61

8. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

9. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

(A) The second height is 10% less than the first.

(B) The first height is 10% more than the second.

(C) The second height is 21% less than the first.

(D) The first height is 21% more than the second.

(E) The second height is 80% of the first.

10. How many rearrangements of *abcd* are there in which no two adjacent letters are also adjacent letters in the alphabet? For example, no such rearrangements could include either *ab* or *ba*.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. The ratio of the length to the width of a rectangle is $4 : 3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

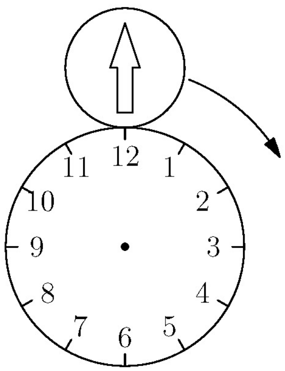
12. Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

(A) 1 **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\sqrt{1+\pi}$ **(E)** $1 + \sqrt{\pi}$

13. Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

14. The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



(A) 2 o'clock **(B)** 3 o'clock **(C)** 4 o'clock **(D)** 6 o'clock **(E)** 8 o'clock

15. Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

16. If $y+4=(x-2)^2$, $x+4=(y-2)^2$, and $x \neq y$, what is the value of x^2+y^2?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

17. A line that passes through the origin intersects both the line $x=1$ and the line $y=1+\frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

(A) $2\sqrt{6}$ **(B)** $2+2\sqrt{3}$ **(C)** 6 **(D)** $3+2\sqrt{3}$ **(E)** $6+\frac{\sqrt{3}}{3}$

18. Hexadecimal (base-16) numbers are written using the numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

19. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\frac{5\sqrt{2}}{3}$ **(B)** $\frac{50\sqrt{3}-75}{4}$ **(C)** $\frac{15\sqrt{3}}{8}$ **(D)** $\frac{50-25\sqrt{3}}{2}$ **(E)** $\frac{25}{6}$

20. A rectangle has area A cm^2 and perimeter P cm, where A and P are positive integers. Which of the following numbers cannot equal $A+P$?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

21. Tetrahedron $ABCD$ has $AB=5$, $AC=3$, $BC=4$, $BD=4$, $AD=3$, and $CD=\frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

22. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\frac{47}{256}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{49}{256}$ **(D)** $\frac{25}{128}$ **(E)** $\frac{51}{256}$

23. The zeros of the function $f(x)=x^2-ax+2a$ are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18